SHOW ALL THE STEPS TO GET FROM

$$\left.\frac{dU(x_1)}{dx_1}\right|_{x_1=x_0} = \frac{d}{dx_1}\left\{m_1 g x_1 - m_2 g\sqrt{\left(\frac{b-x_1}{2}\right)^2 - d^2}\right\} = 0$$

TO

$$x_0 = b - \frac{4m_1 d}{\sqrt{4m_1^2 - m_2^2}}$$

$$\frac{dU(x_1)}{dx_1} = -m_1 g - m_2 g\left\{\frac{1}{2}\left[\left(\frac{b-x_1}{2}\right)^2 - d^2\right]^{-\frac{1}{2}}\left[2\left(\frac{b-x_1}{2}\right)\left(-\frac{1}{2}\right)\right]\right\} = 0$$

$$\Rightarrow m_1 g = m_2 g\left\{\frac{-(b-x_0)}{4\sqrt{\left(\frac{b-x_0}{2}\right)^2 - d^2}}\right\} \quad \text{(} x_1 \to x_0 \text{ WHEN SET TO ZERO)}$$

$$4m_1\sqrt{\left(\frac{b-x_0}{2}\right)^2 - d^2} = -m_2(b-x_0)$$

ISOLATE RADICAL & SQUARE BOTH SIDES

$$\left(\frac{b-x_0}{2}\right)^2 - d^2 = \left(\frac{m_2}{4m_1}\right)^2 (b-x_0)^2$$

FACTOR OUT $(b-x_0)^2$

$$(b-x_0)^2\left[\frac{1}{4} - \frac{m_2^2}{16m_1^2}\right] = d^2$$

$$(b-x_0)^2\left(\frac{4m_1^2 - m_2^2}{16m_1^2}\right) = d^2$$

TAKING THE SQUARE ROOT GIVES

$$b - x_0 = \frac{4m_1 d}{\sqrt{4m_1^2 - m_2^2}}$$

$$\Rightarrow \boxed{x_0 = b - \frac{4m_1 d}{\sqrt{4m_1^2 - m_2^2}}} \quad \text{QED!}$$